Eidgenössische Technische Hochschule Zürich
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Computational Quantum Physics
Exercise 12

FS 16
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The goal of this exercise is to compare the performance of simulated thermal and quantum annealing for finding the ground state energy of a classical 2D spin glass

$$H = -\sum_{\langle i,j \rangle} J_{i,j} s_i s_j ,$$

where the spins are located on a square lattice with open boundary conditions and coupled to their nearest neighbors. In the file `instance.txt` on moodle, you are provided with a randomly-generated problem instance containing the linear lattice size L, nearest-neighbor couplings $J_{i,j} \in \{\pm 1\}$, as well as the exact ground state energy E (the file format is described in `readme.txt`).

Problem 12.1 Simulated Thermal Annealing

For the exercise sheet 10 you implemented a Monte Carlo simulation of the classical 2D Ising model on a square lattice using local updates.

- Adapt your code to use open boundary conditions and the provided couplings $J_{i,j}$.
- To simulate thermal annealing, run the Monte Carlo simulation and lower the temperature from $\beta = 0.1$ to $\beta = 5$ by $\Delta\beta = 10^{-5}$ after each Monte Carlo sweep. Once you reached $\beta = 5$, compare the energy of the Monte Carlo configuration with the exact energy provided to you.
- Run the simulation 100 times and calculate the success rate, i.e., the fraction of runs for which the correct ground state energy was obtained.

Problem 12.2 Simulated Quantum Annealing

For the last exercise sheet you implemented a quantum Monte Carlo simulation for the quantum 1D Ising model in a transverse field by mapping it onto a classical 2D Ising model.

- Implement a quantum Monte Carlo simulation for the 2D model

$$H = -\alpha \sum_{\langle i,j \rangle} J_{i,j} \sigma_i^z \sigma_j^z - \Gamma \sum_i \sigma_i^x$$

  with the provided couplings $J_{i,j}$ and open boundary conditions. Choose the time discretization $\Delta\tau$ such that the corresponding classical problem – which is (2+1)-dimensional! – has the same number of sites in each dimension.
- To simulate quantum annealing, run the quantum Monte Carlo simulation at fixed temperature β, starting with $\alpha = 0$ and $\Gamma = 1$. After each sweep, increase the coupling strength $\alpha = s$ and decrease the transverse magnetic field $\Gamma = 1 - s$ by some Δs, until you reach $\alpha = 1$ and $\Gamma = 0$.
- Run the simulation 1000 times and calculate the success rate (as above). Explore its dependence on the inverse temperature β and on Δs.